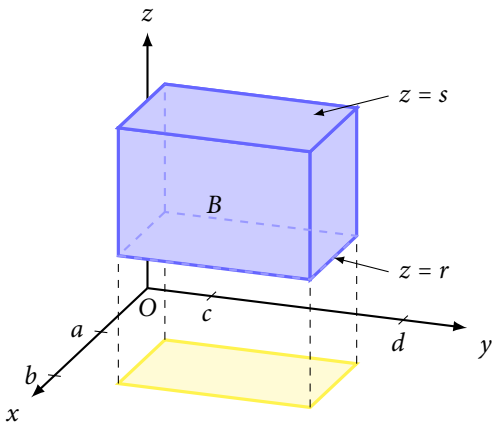
Lesson 27b. Triple Integrals, cont.

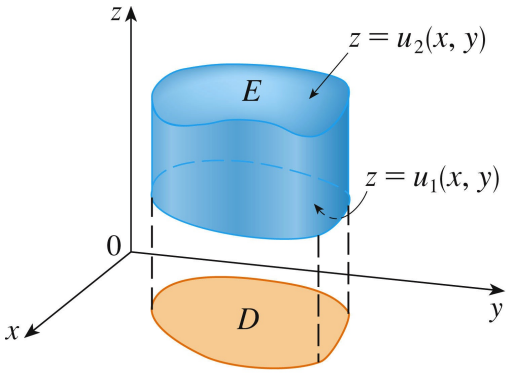
1 Last time...

Rectangular boxes



$$\iiint_B f(x,y,z)\,dV = \int_a^b \int_c^d \int_r^s f(x,y,z)\,dz\,dy\,dx$$

Type A 3D regions

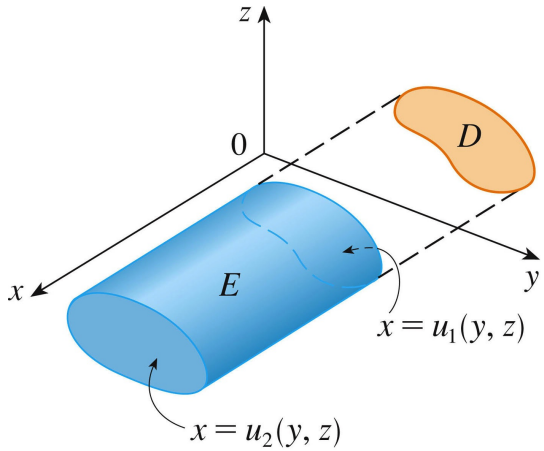


$$\iiint_E f(x,y,z)\,dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz \right] dA$$

Example 1. Express $\iiint_E y\sqrt{z}\,dV$ as an iterated integral, where E is the solid tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$.

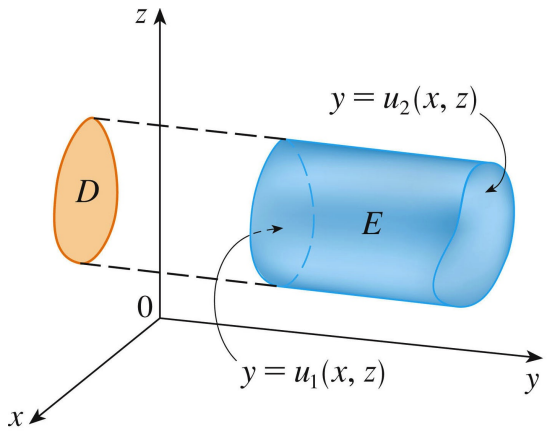
2 Integrating over other types of 3D regions

- **Type B 3D region:** between two continuous functions of y and z



Example 2. Express $\iiint_E y\sqrt{z}\,dV$ as an iterated integral, where E is the tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$. Consider E as a type B region.

- **Type C 3D region:** between two continuous functions of x and z



Example 3. Express $\iiint_E y\sqrt{z}\,dV$ as an iterated integral, where E is the tetrahedron enclosed by the coordinate planes and the plane $2x + y + z = 4$. Consider E as a type C region.

3 If we have time...

Example 4. Express $\iiint_E \sin(x + yz)\, dV$ as an iterated integral, where E lies below the surface $z = 1 + x^2 + 4y^2$ and above the region in the xy-plane bounded by the curves $x = 2y$, $x = 0$, and $y = 1$.